"Back-of-the-envelope" wind and altitude correction for 100 metre sprint times

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Abstract

A simple algebraic expression is presented to correct men's and women's 100 metre sprint times for ambient wind and altitude effects. The simplified formula is derived from a more complicated equation of motion used to model the athlete's velocity as a function of time (the velocity curve). This method predicts adjustments to 0-wind and 0-altitude equivalents, which are in excellent agreement to other estimates presented in the literature. The expression is easily programmable on any computer, and could conveniently be used by coaches, meet directors, or the media to assess the performance of athletes and the quality of a race immediately following the event.

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1 Introduction

Although not officially recognized by the International Amateur Athletic Federation (IAAF), correcting sprint race times for the effects of wind and altitude variation is a subject of increasing interest in the Track and Field community. With the number of men's sub-9.90 s and women's sub-10.80 s clockings on the rise, correcting these marks to their sea-level, 0-wind equivalents is useful in determining the overall quality of the performances (at the time of competition). A literature search reveals rather detailed experimental field studies of these effects [1, 2, 3, 4, 5], as well as several theoretical estimates based on mathematical and computational simulations [6, 7, 8].

Physically, linear drag forces (scaled to units of mass) are expressed as

$$F_d = \frac{1}{2} C_d A \rho(H)(v(t) - w)^2 \ , \tag{1}$$

where C_d is the drag coefficient, A the frontal cross-sectional area, $\rho(H)$ the atmospheric density at altitude H, $v(t)$ the sprinter's velocity, and w the wind speed (co-linear to $v(t)$). It follows that F_d will be smaller for tail-winds ($w > 0$), and larger for head-winds ($w < 0$) at a fixed altitude. Head-winds and tail-winds of equal magnitude will not provide equal-magnitude time corrections, due to the non-linear nature of the drag term.

At 0 metres altitude with 0 wind, the base drag is $F_0 = 1/2 C_d A \rho_0 v(t)^2$, where $\rho_0 = 1.184$ g cm^{-3} is the mean sea-level density of air at 25 degrees Celsius. Since air density varies exponentially as a function of altitude, a convenient approximation can be written as $\rho(H) = \rho_0 \exp(-0.000125 \cdot H)$ [1] for the range of elevations considered herein (less than 2300 m for the majority of competition venues).

The general consensus of most researchers in question is that for a 10.00 s race (average men's world-class sprint), a tail-wind of +2 ms^{-1} will provide an advantage of roughly 0.10 seconds (*i.e.* a faster time), whose value will vary slightly depending on the altitude of the competition venue. If the wind gauge reads in excess of +2 ms^{-1}, the performance is termed wind-assisted, and is not eligible for any potential record status. Conversely with no wind, an altitude of 1000 m will produce an advantage of 0.03 s, above which performances are officially deemed altitude-assisted. Unlike wind-assisted marks, an altitude-assisted time can still count for a record. At 2000 m, the advantage will be about 0.06 s over a sea-level run. An 11.00 s time (average world-class women) will be boosted by about +0.12 s with a +2 ms^{-1} tail-wind, and by 0.07 s (no wind) at 2000 m. As altitude increases, the magnitude of the wind effects will increase. Obviously, this is

a reasonable explanation for the rash of World Records (WRs) experienced in the sprints and long jump at the 1968 Olympics in Mexico City, which resides at an altitude of approximately 2250 m.

2 "Back-of-the-Envelope" correction: Derivation

A "back-of-the-envelope" (BOTE) calculation is a simplified reduction of a complex (physical) model, from which one can make reasonable predictions with a minimal number of input parameters. An exact modeling of wind and altitude effects is a daunting task, since the mechanics involved are numerous and not easily representable by basic functions (see [7] for such a model). A historically-based method of such simulations is via the velocity curve approach. This is a method of studying a sprinter's performance, first introduced empirically by Hill [9] in the early 1900s, and further investigated by Keller [10] as an equation of motion of the form

$$\dot{v}(t) = F_p - v(t)\ \alpha^{-1}\ , \tag{2}$$

The term F_p is a propulsive term, while α is a decay term (representing internal physiological or biomechanical variables). Note that again Equations (2) is scaled in units of the sprinter's mass M, so the interpretation of F_d is force per unit mass (or, effectively, acceleration). Unless otherwise specified, this notation is used for the remainder of the article.

This derivation roughly follows that of Reference [8], however the latter incorrectly estimates the numerical value of certain key variables, and omits the effects of altitude all together. In fact, the author of [8] suggests that altitude effects on sprint times *cannot* be modeled by drag modification alone, which is not necessarily a correct assertion (as will be shown).

Equation (2) may easily be altered to include drag effects by the addition of F_d,

$$\dot{v}(t) = F_p - v(t)\ \alpha^{-1} - F_d\ , \tag{3}$$

and a time dependence $F_p \to F_p(t)$ may also be added (see *e.g.* [7, 11, 12] for such mechanisms). The BOTE expression presented herein, being simplistic by its namesake, does not include these substitutions, and furthermore imposes an additional simplification of time-independence, $v(t) \to v$, and $\dot{v}(t) = 0$.

To address the issue of wind and altitude correction, define as follows $v(w, H)$ (velocity of the sprinter at altitude H with wind w); $v(0, 0)$ (velocity with 0-wind, at sea-level); and $F_d(w, H)$ (effective drag for wind w and

altitude H). Subject to the constraint of constant velocity, Equation (3) may be rewritten as

$$\begin{aligned} v(w,H) &= \alpha[F_p - F_d(w,H)] \ , \\ v(0,0) &= \alpha[F_p - F_d(0,0)] \ , \end{aligned} \tag{4}$$

for each case described above.

Solving for α and equating the two expressions above yields

$$\frac{v(0,0)}{v(w,H)} = \frac{(1 - F_d(0,0)/F_p)}{(1 - F_d(w,H)/F_p)} \ , \tag{5}$$

Define the ratio $\delta = F_d(0,0)/F_p$ as the effort required to overcome drag in 0-wind conditions at sea level. The numerical value of δ will be discussed shortly.

Since velocity is constant (*i.e.* the average race velocity), one can write $v(w,H) = 100/t_{w,H}$, where $t_{w,H}$ is the official time for the race under consideration, and rewrite Equation (5) as

$$\frac{t_{0,0}}{t_{w,H}} = \frac{(1 - F_d(w,H)/F_p)}{(1-\delta)} \ , \tag{6}$$

To simplify this expression further, note that the drag force for arbitrary w and H can be written as

$$\begin{aligned} F_d(w,H) &= \frac{1}{2} C_d A \rho(H) v(w,H)^2 \left(1 - \frac{w}{v(w,H)}\right)^2 \\ &= F_d(0,0) \left(\frac{v(w,H)}{v_{0,0}}\right)^2 \exp(-0.000125 \cdot H) \left(1 - \frac{w \cdot t_{w,H}}{100}\right)^2 \end{aligned} \tag{7}$$

So, replacing $(v(w,H)/v_0) = (t_{0,0}/t_{w,H})$,

$$\frac{F_d(w,H)}{F_p} = \delta \left(\frac{t_{0,0}}{t_{w,H}}\right)^2 \exp(-0.000125 \cdot H) \left(1 - \frac{w \cdot t_{w,H}}{100}\right)^2 \ , \tag{8}$$

and thus

$$\frac{t_{0,0}}{t_{w,H}} = \frac{1}{(1-\delta)} \left[1 - \delta \left(\frac{t_{0,0}}{t_{w,H}}\right)^2 exp(-0.000125 \cdot H) \left(1 - \frac{w \cdot t_{w,H}}{100)}\right)^2\right] \ . \tag{9}$$

Unfortunately, this is now a quadratic expression in $(t_{0,0}/t_{w,H})$, but this problem is quickly resolved by making the following substitution. Rewrite $(t_{0,0}/t_{w,H}) = 1 + \Delta t/t_{w,H}$, with $\Delta t = t_{0,0} - t_{w,H}$. Since Δt will seldom be larger than 0.3 s for a ∼10 s race, it is reasonable to make the substitution

$$\left(\frac{t_{0,0}}{t_{w,H}}\right)^2 = \left(1 + \frac{\Delta t}{t_{w,H}}\right)^2 \simeq 1 + 2\,\frac{\Delta t}{t_{w,H}} = 2\,\frac{t_{0,0}}{t_{w,H}} - 1 \tag{10}$$

The numerical value of δ is determined as

$$\delta = \frac{F_d(0,0)}{F_p} = \frac{1}{2}\frac{C_d A \rho_0 v^2}{M\ F_p} \tag{11}$$

(recall that the earlier definition of F is scaled in units of inverse mass, hence the need for M in the denominator). Pritchard [8] initially found a value of $\delta \simeq 0.032$ (i.e. 3.2% of a sprinter's effort is required to overcome drag), however this assumed an overestimated value of the drag coefficient $C_d = 1.0$, as well as the mean propulsive force $F_p = 12.1$ ms^{-2}. Current research suggests a drag coefficient of $C_d \in (0.5, 0.6)$ [7, 13], as well as an average $F_p \sim 7$ ms^{-2} [7, 11, 12].

For a $9.90 - 10.00$ s race, the average velocity is between $v = 10 - 10.1$ m s^{-1}. Taking the drag area to be $C_d \cdot A = 0.23$ m^2 (consistent with the quoted C_d values, and cross-sectional area $A \in (0.4, 0.5)$ m^2, for a sprinter of mass 75 kg, one finds $\delta \sim 0.027$.

Since δ is small, $1/(1-\delta) \simeq (1+\delta)$, and including the approximation of Equation (10), Equation (9) may be rearranged as

$$\begin{aligned}\frac{t_{0,0}}{t_{w,H}} &= \left(\frac{1}{1-\delta}\right)\left[1 - \delta\exp(-0.000125\cdot H)\left(2\,\frac{t_{0,0}}{t_{w,H}} - 1\right)\left(1 - \frac{w\cdot t_{w,H}}{100}\right)^2\right] \\ &\simeq 1 + \delta - \delta\exp(-0.000125\cdot H)\left(1 - \frac{w\cdot t_{w,H}}{100}\right)^2 + o(\delta^2)\ , \end{aligned} \tag{12}$$

Thus, inserting the numerical value of δ, one obtains the "back-of-the-envelope" calculation

$$t_{0,0} \simeq t_{w,H}[1.027 - 0.027\exp(-0.000125\cdot H)(1 - w\cdot t_{w,H}/100)^2]\ . \tag{13}$$

For women, the input parameters F_p, A, v^2, and M are smaller, so assuming values $v = (100/11) = 9.1$ ms^{-1}, $M = 65$ kg, $F_p \sim 5$ ms^{-2}, and $A \sim 0.35$ m^2, δ remains essentially unchanged.

Equation (13) provides an excellent match to the predictions of Reference [7], as well as those of Dapena [2] and Linthorne [5]. Thus, 100 metre

sprint times may be corrected to their 0-wind, sea level equivalents by inputting only the official time, the wind gauge reading, and the altitude of the sporting venue. Furthermore, Equation (8) is easily programmable in most scientific calculators and personal computers, and hence may be used track-side by coaches, officials and the media immediately following a race to gauge its overall "quality".

3 Applications

To demonstrate the utility of Equation (13), Tables 1, 2, 3, and 4 present the corresponding corrections to the top five all-time men's and women's 100 m performances run with legal tail-winds, illegal winds ($w > +2.0$ ms^{-1}), altitude effects ($H > 1000$ m), and extreme head-winds ($w < -1$ ms^{-1}).

The current 100 m World Record (as of June 2000) of 9.79 s by Maurice Greene was run at low altitude with virtually no wind, and adjusts to 9.80 s. Note that (Table 1) the 9.86 s performances of Trinidadian Ato Boldon and Namibia's Frank Fredericks were both run into head-winds of equal magnitude, but the altitude difference allows for a 0.02 s differential in corrected times. The former World Record of 9.84 s by Canada's Donovan Bailey corrects to a 9.88 s. It is also interesting to note how exceptional performances can be hampered by strong head-winds (Table 4).

The 10.49 s WR of the late Florence Griffith-Joyner is included in both Table 1 and Table 2, to demonstrate the common belief that this mark was strongly wind-aided (despite the fact that the official wind gauge reading was +0.0 ms^{-1}, there is strong circumstantial evidence to suggest that the equipment malfunctioned). Griffith-Joyner's legal personal records (PRs) correct to about 10.68 s, while her +2.1 ms^{-1} wind-aided 10.54 s (Seoul, 1988) corrects to 10.66 s. Thus, the actual WR mark should probably be 10.60-10.65 s (or a 0-wind, 0-altitude equivalent of about 10.66-10.68 s). American Marion Jones' current adjusted PR is 10.69 s, effectively on par with Griffith-Joyner's best marks.

From a historical perspective, it is interesting to note that Calvin Smith's 10.04 s (-2.2 ms^{-1}) in 1983 would have converted to a 0-wind 9.93 s at sea level. Smith's actual WR of 9.93 s (+1.2 ms^{-1}) was run at altitude (Colorado Springs, USA; 1850 m), correcting to only 10.03 s. The former WRs of Canada's Ben Johnson, 9.79 s (+1.1 ms^{-1}) in Seoul, SKR, and 9.83 s (+1.0 ms^{-1}) in Rome, ITA, would correct to 9.85 s and 9.88 s, respectively. Unfortunately, these marks were stricken as a result of performance-enhancing drug infractions.

4 Conclusions

The presented "back-of-the-envelope" calculation is simple to use, and is applicable to both men and women's performances. An on-line JavaScript version is available at the author's website, currently

`http://palmtree.physics.utoronto.ca/~newt/track/wind/`

It is hoped that its use may be eventually adopted by the IAAF and/or other governing bodies of Athletics as a relative gauge of performance quality under differing competition conditions.

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Athlete	$t_{w,H}$ (w)	Venue (Altitude)	Date	$t_{0,0}$
Maurice Greene USA	9.79 (+0.1)	Athens, GRE (110)	99/06/16	9.80
Bruny Surin CAN	9.84 (+0.2)	Athens, GRE	99/08/22	9.85
Donovan Bailey CAN	9.84 (+0.7)	Atlanta, USA (315)	96/07/27	9.88
Leroy Burrell USA	9.85 (+1.2)	Lausanne, SWI (600)	94/07/06	9.92
Ato Boldon TRI	9.86 (−0.4)	Athens, GRE	98/06/17	9.84
Frank Fredericks NAM	9.86 (−0.4)	Lausanne, SWI	96/07/03	9.86
Florence Griffith-Joyner USA	10.49 (+0.0)	Indianapolis, USA (220)	88/07/16	10.50
	10.61(+1.2)	Indianapolis, USA	88/07/17	10.68
Marion Jones USA	10.72 (+0.0)	Monaco (10)	98/08/08	10.72
Christine Aaron FRA	10.73 (+2.0)	Budapest, HUN (150)	98/08/19	10.84
Merlene Ottey JAM	10.74 (+1.3)	Milano, ITA (121)	96/09/07	10.82
Evelyn Ashford USA	10.76 (+1.7)	Zurich, SWI (410)	84/08/22	10.87

Table 1: Men's and Women's top 5 all-time legal 100 m performances (best per athlete). Times measured in seconds (s), and wind-speeds in ms^{-1}. Altitude is assumed to be correct to within ± 20 m.

Athlete	$t_{w,H}$ (w)	Venue (Altitude)	Date	$t_{0,0}$
Obadele Thompson BAR	9.69 (+5.7)	El Paso, USA (1300)	96/04/13	9.91
Carl Lewis USA	9.78 (+5.2)	Indianapolis, USA	88/07/16	9.98
Andre Cason USA	9.79 (+4.5)	Eugene, USA	93/06/16	9.97
Maurice Greene USA	9.79 (+2.9)	Eugene, USA	98/05/31	9.92
Leonard Scott USA	9.83 (+7.1)	Knoxville, USA (270)	99/04/09	10.07
Griffith-Joyner USA	10.49 (+5.5)	Indianapolis, USA	88/07/16	10.72
	10.54(+3.0)	Seoul, SKR (85)	88/09/25	10.69
Marion Jones USA	10.75 (+4.1)	New Orleans, USA (10)	98/06/19	10.95
Gail Devers USA	10.77 (+2.3)	San Jose, USA (10)	94/05/28	10.90
Ekaterini Thanou GRE	10.77 (+2.3)	Rethymno, GRE (20)	99/05/29	10.90
Evelyn Ashford USA	10.78 (+3.1)	Modesto, USA (25)	84/05/12	10.94

Table 2: Top 5 all-time wind-assisted marks ($w > +2.0$ ms^{-1}).

Athlete	$t_{w,H}$ (w)	Venue (Altitude)	Date	$t_{0,0}$
Obadele Thompson BAR	9.87 (−0.2)	Johannesburg, RSA (1750)	98/09/11	9.91
Seun Ogunkoya NGR	9.92 (−0.2)	Johannesburg, RSA	98-09-11	9.96
Calvin Smith USA	9.93 (+1.4)	Colorado Springs, USA (1853)	83/07/03	10.04
Jim Hines USA	9.95 (+0.3)	Mexico City, MEX (2250)	68/10/14	10.03
Olapade Adeniken NGR	9.95 (+1.9)	El Paso, USA (1300)	94-04-16	10.07
Marion Jones USA	10.65 (+1.1)	Johannesburg, RSA	98/09/12	10.76
Dawn Sowell USA	10.78 (+1.0)	Provo, USA (1380)	89/06/03	10.81
Evelyn Ashford USA	10.79 (+0.6)	Colorado Springs, USA	83/07/03	10.88
Diane Williams USA	10.94 (+0.6)	Colorado Springs, USA	83/07/03	11.03
Chandra Sturrup BAH	10.97 (+1.1)	Johannesburg, RSA	98/09/12	11.08

Table 3: Top 5 all-time altitude-assisted marks ($H > 1000$ m).

Athlete	$t_{w,H}$ (w)	Venue (Altitude)	Date	$t_{0,0}$
Maurice Greene USA	9.96 (-1.0)	Uniondale, USA (30)	98/07/21	9.92
Leroy Burrell USA	9.97 (-1.3)	Barcelona, ESP (95)	92/08/01	9.90
Linford Christie USA	10.00 (-1.3)	Barcelona, ESP	92/08/01	9.93
Ato Boldon TRI	10.00 (-1.0)	Uniondale, USA	98/07/21	9.96
Donovan Bailey CAN	10.03 (-2.1)	Abbotsford, CAN (40)	97/07/19	9.91
Irena Privalova USR	10.84 (-1.0)	Barcelona, ESP	92/08/11	10.77
Gwen Torrence USA	10.86 (-1.0)	Barcelona, ESP	92/08/01	10.79
Merlene Ottey JAM	10.88 (-1.0)	Barcelona, ESP	92/08/01	10.81
Evelyn Ashford USA	10.96 (-1.4)	Knoxville, USA	82/06/19	10.88
Jones USA	10.97 (-1.1)	Indianapolis, USA	97/06/13	10.90

Table 4: Top 5 all-time marks with $w \leq -1.0$ ms^{-1}.